Towards quantum superpositions of a mirror: an exact open systems analysis—calculational details

2005 J. Phys. A: Math. Gen. 38 2715

(http://iopscience.iop.org/0305-4470/38/12/013)

INSTITUTE OF PHYSICS PUBLISHING — JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **38** (2005) 2715–2727 doi:10.1088/0305-4470/38/12/013

Towards quantum superpositions of a mirror: an exact open systems analysis—calculational details

Stephen L Adler[1], **Angelo Bassi**[2,3] **and Emiliano Ippoliti**[3,4]

[1] Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540 USA
[2] The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy
[3] Istituto Nazionale di Fisica Nucleare, sezione di Trieste, Trieste, Italy
[4] Department of Theoretical Physics, University of Trieste, Strada Costiera 11, 34014 Trieste, Italy

E-mail: adler@ias.edu, bassi@ictp.trieste.it and ippoliti@ts.infn.it

Received 22 December 2004, in final form 30 December 2004
Published 9 March 2005
Online at stacks.iop.org/JPhysA/38/2715

Abstract
We give details of calculations analysing the proposed mirror superposition experiment of Marshall, Simon, Penrose and Bouwmeester within different stochastic models for state vector collapse. We give two methods for exactly calculating the fringe visibility in these models, one proceeding directly from the equation of motion for the expectation of the density matrix, and the other proceeding from solving a linear stochastic unravelling of this equation. We also give details of the calculation that identifies the stochasticity parameter implied by the small-displacement Taylor expansion of the CSL model density matrix equation. The implications of the two results are briefly discussed. Two pedagogical appendices review mathematical apparatus needed for the calculations.

PACS numbers: 03.65.Ta, 03.65.Yz, 05.40.−a

1. Introduction

There is currently much interest in experiments to create quantum superposition states involving large numbers of particles, with the ultimate aim of testing whether quantum superpositions of macroscopic systems can be observed. Recently, Marshall, Simon, Penrose and Bouwmeester [1], motivated by suggestions of Penrose [2], have proposed a novel interferometric experiment in which a single photon interacts with a miniature mirror mounted on a cantilever in one arm of the interferometer, thus setting up a superposition of states containing of order 10^{14} atoms. Since the two superposed states in this experiment have a relative centre-of-mass displacement of order the width of the mirror centre-of-mass wave packet $\sim\sigma \sim 10^{-11}$ cm, the experiment will place new constraints on proposals for modifications to quantum mechanics in which the centre-of-mass displacement is the key parameter.

Among the different proposals, collapse models [3–6] have been extensively studied. The basic idea is to combine the standard Schrödinger evolution and the postulate of wavepacket reduction into one universal dynamical equation, which is assumed to govern all physical processes. Such a dynamics accounts both for the quantum properties of microscopic systems and for the classical properties of macroscopic ones; in particular, it guarantees that measurements made on microscopic systems always have definite outcomes, and with the correct quantum probabilities (Born probability rule).

In a recent letter [7], we have analysed the Marshall *et al* experiment within the framework of the GRW [3], CSL [4] and QMUPL [5] models, and have shown that—within the CLS model, which predicts the largest deviation from standard quantum predictions—one expects the maintenance of coherence to better than 1 part in 10^8. Our aim in this paper is to give the derivations of formulae presented, without derivation, in our letter. In section 2 we give the basic equations for the Marshall *et al* experiment, first as formulated in their paper, and then as formulated within the collapse models. In section 3 we solve for the visibility (the physical quantity measured in the experiment) by direct calculation from the density matrix evolution equation, making use of the interaction picture, the Baker–Hausdorff formula, and cyclic permutation under a trace. In section 4 we give an alternative derivation of the visibility, obtained by solving a linear stochastic unravelling of the density matrix equation, using the Itô stochastic calculus. In section 5 we compute the stochasticity parameter entering into the visibility formula in terms of the parameters of the CSL model. We briefly summarize our results and their application to the Marshall *et al* experiment in section 6. In appendix A we derive the Baker–Hausdorff formula used in the text, and in appendix B we review the Itô calculus formulae used in the calculation of section 4.

2. Basic formalism

The Hamiltonian for the Marshall *et al* experiment, with the moving mirror in a cavity in interferometer arm A, is [8]

$$H = \hbar\omega_c\left(a_A^\dagger a_A + a_B^\dagger a_B\right) + \hbar\omega_m b^\dagger b - \hbar G a_A^\dagger a_A (b + b^\dagger). \tag{1}$$

Here ω_c is the frequency of the photon, $a_A^\dagger$ and $a_B^\dagger$ are the creation operators for the photon in the interferometer arms A and B, respectively, while ω_m and $b^\dagger$ are the frequency and the phonon creation operator associated with motion of the centre of mass of the mirror. The coupling constant is $G = \omega_c\sigma/L$, where L is the length of the cavity, with $\sigma = (\hbar/2M\omega_m)^{\frac{1}{2}}$ the width of the mirror wave packet and M the mass of the mirror.

The semi-silvered beam splitter of the interferometer places the photon in an initial state that is an equal superposition of being in arm A or B,

$$|\psi_0\rangle = \tfrac{1}{\sqrt{2}}[|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B]|0\rangle_m, \tag{2}$$

and standard quantum mechanics predicts that at time t the state vector will be

$$|\psi_t\rangle = \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}Ht}|\psi_0\rangle = \tfrac{1}{\sqrt{2}}\,\mathrm{e}^{-\mathrm{i}\omega_c t}[|0\rangle_A|1\rangle_B|0\rangle_m + \mathrm{e}^{\mathrm{i}\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_A|0\rangle_B|\alpha_t\rangle_m]. \tag{3}$$

Here we have written $\kappa = G/\omega_m$ and $|\alpha_t\rangle_m$ denotes a unit normalized mirror coherent state with complex amplitude $\alpha_t = \kappa(1 - \mathrm{e}^{-\mathrm{i}\omega_m t})$. While in state $|0\rangle_m$ the mirror is fixed at its equilibrium position (the origin of the reference frame), in state $|\alpha_t\rangle_m$ the mirror oscillates between 0 and $\ell \equiv 4\kappa\sigma$; in both cases, the shape of the wavefunction (in position) is a Gaussian of width σ.

The physically measurable quantity considered by Marshall *et al* is the maximum interference visibility for the photon $\nu(t)$, defined as twice the modulus of the off-diagonal element of the reduced density matrix of the photon. The full density matrix for the system is

$$\begin{aligned}\rho &= |\psi_t\rangle\langle\psi_t| \\ &= \tfrac{1}{2}[|0\rangle_{AA}\langle 0||1\rangle_{BB}\langle 1||0\rangle_{mm}\langle 0| + |1\rangle_{AA}\langle 1||0\rangle_{BB}\langle 0||\alpha_t\rangle_{mm}\langle\alpha_t| \\ &\quad + \mathrm{e}^{\mathrm{i}\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_{AA}\langle 0||0\rangle_{BB}\langle 1||\alpha_t\rangle_{mm}\langle 0| \\ &\quad + \mathrm{e}^{-\mathrm{i}\kappa^2(\omega_m t - \sin\omega_m t)}|0\rangle_{AA}\langle 1||1\rangle_{BB}\langle 0||0\rangle_{mm}\langle\alpha_t|].\end{aligned} \tag{4}$$

Thus after tracing over the mirror states, the reduced density matrix has as the coefficient of the off-diagonal term $|1\rangle_{AA}\langle 0||0\rangle_{BB}\langle 1|$ the factor $\frac{1}{2}f$, with

$$f = \mathrm{e}^{\mathrm{i}\kappa^2(\omega_m t - \sin\omega_m t)}{}_m\langle 0|\alpha_t\rangle_m, \tag{5}$$

which using ${}_m\langle 0|\alpha_t\rangle_m = \mathrm{e}^{-\frac{1}{2}|\alpha_t|^2}$ gives

$$f = \mathrm{e}^{\mathrm{i}\kappa^2(\omega_m t - \sin\omega_m t)}\,\mathrm{e}^{-\kappa^2(1-\cos\omega_m t)}. \tag{6}$$

Thus, under standard quantum mechanical evolution of the state, one has for the time dependence of the visibility

$$\nu(t) = \mathrm{e}^{-\kappa^2(1-\cos\omega_m t)}. \tag{7}$$

According to the above formula, the visibility starts from its maximal value 1; it then decreases, but after half a period of the mirror's motion it increases again, reaching the maximal value after one period $T = 2\pi/\omega_m$. The strategy to test the macroscopic superposition of the mirror then goes as follows. One measures the photon's visibility after one period T: if it is close to 1, then no collapse of the mirror's wavefunction has occurred; if on the contrary it is smaller than 1, a spontaneous collapse process is present which reduces the superposition to one of its two terms. Of course, one must keep control of all sources of decoherence, which tend to lower the observed visibility.

We proceed now to reanalyse the experiment using the modified Schrödinger evolution of the QMUPL model of wavefunction collapse [5]; this model is particularly useful since, as we shall prove, it allows one to get an exact formula for the visibility when a spontaneous collapse mechanism is present. Moreover, this model corresponds to the leading term in the small-displacement Taylor expansion of both the GRW and the CSL models; such an expansion is particularly suitable to the present case since, according to the parameters of the experiment, the maximum displacement between the two superposed states of the mirror is of order 10^{-11} cm, which is much smaller than the typical distance of 10^{-5} cm required for quantum superpositions to be destroyed, in the GRW and CSL models. Under the QMUPL model, the state vector evolves as

$$\mathrm{d}|\psi_t\rangle = \left[-\frac{\mathrm{i}}{\hbar}H\,\mathrm{d}t + \sqrt{\eta}(q - \langle q\rangle_t)\,\mathrm{d}W_t - \frac{\eta}{2}(q - \langle q\rangle_t)^2\,\mathrm{d}t\right]|\psi_t\rangle, \tag{8}$$

where H is given by equation (1), and $\langle q\rangle_t \equiv \langle\psi_t|q|\psi_t\rangle$ is the quantum-mechanical expectation of the position operator $q = \sigma(b + b^\dagger)$ associated with the centre of mass of the mirror. The stochastic dynamics is governed by a standard Wiener process W_t, defined on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$. Using the rules of the Itô calculus (see appendix B), the density matrix evolution corresponding to equation (8) is

$$\mathrm{d}\hat{\rho} = -\frac{\mathrm{i}}{\hbar}[H, \hat{\rho}]\,\mathrm{d}t - \frac{1}{2}\eta[q, [q, \hat{\rho}]]\,\mathrm{d}t + \sqrt{\eta}[\hat{\rho}, [\hat{\rho}, q]]\,\mathrm{d}W_t. \tag{9}$$

Since to observe interference fringes experimentally requires passing to an ensemble of identically prepared photons through the apparatus, the relevant density matrix in the stochastic case is the ensemble expectation $\rho = E[\hat{\rho}]$, which obeys the ordinary differential equation

$$\begin{aligned}\frac{d\rho}{dt} &= -\frac{i}{\hbar}[H, \rho] - \frac{1}{2}\eta[q, [q, \rho]] \\ &= -\frac{i}{\hbar}[H, \rho] - \frac{1}{2}\eta\sigma^2[b + b^\dagger, [b + b^\dagger, \rho]].\end{aligned} \tag{10}$$

Defining an off-diagonal density matrix ρ_{OD} acting in the mirror Hilbert subspace by ${}_A\langle 1|_B\langle 0|\rho|0\rangle_A|1\rangle_B = \frac{1}{2}\rho_{OD}$, so that the factor f introduced above is $\mathrm{Tr}_m \rho_{OD}$, we can project out from equation (10) the evolution equation for ρ_{OD},

$$\frac{d\rho_{OD}(t)}{dt} = -iH^A\rho_{OD}(t) + i\rho_{OD}(t)H^B - \frac{1}{2}\eta\sigma^2[b + b^\dagger, [b + b^\dagger, \rho_{OD}(t)]], \tag{11}$$

with $\hbar H^A$ the effective mirror Hamiltonian acting when the photon passes through interferometer arm A, and with $\hbar H^B$ the corresponding effective mirror Hamiltonian acting when the photon passes through arm B,

$$H^A = \omega_m b^\dagger b - G(b + b^\dagger) \qquad H^B = \omega_m b^\dagger b. \tag{12}$$

We must now solve the dynamics represented by equations (11) and (12), or equivalently by equation (10), so as to calculate $\mathrm{Tr}_m \rho_{OD}$ and obtain the visibility. Additionally, we must calculate the stochasticity parameter η entering into equations (8)–(11) in terms of the parameters of the CSL model. These are the issues addressed in the following three sections.

3. Direct solution for the visibility from the density matrix evolution equation

In this section we give a calculation of the mean visibility directly from the density matrix equation of motion. An essential identity in everything that follows is the Baker–Hausdorff identity, derived in appendix A,

$$e^{-iH^A t} e^{iH^B t} = N_t\, e^{\alpha_t b^\dagger}\, e^{\beta_t b}, \tag{13}$$

with H^A and H^B as given in equation (12), and with

$$\begin{aligned}N_t &= e^{-\kappa^2(1 - i\omega_m t - e^{-i\omega_m t})} \\ &= e^{-\kappa^2(1-\cos\omega_m t) + i\kappa^2(\omega_m t - \sin\omega_m t)} \\ \alpha_t &= \kappa(1 - e^{-i\omega_m t}) \\ \beta_t &= -\kappa(1 - e^{i\omega_m t}).\end{aligned} \tag{14}$$

Defining the photon off-diagonal part of the density matrix ρ_{OD} as in section 2, which obeys the evolution equation of equation (11), the visibility is $\nu = |\mathrm{Tr}_m \rho_{OD}|$; thus what is needed is to calculate $\mathrm{Tr}_m \rho_{OD}$.

Let us now go to the interaction picture by defining

$$\rho^I_{OD}(t) = e^{iH^A t}\rho_{OD}(t)\, e^{-iH^B t}, \tag{15}$$

so that $\rho^I_{OD}(0) = \rho_{OD}(0) = |0\rangle_{m\,m}\langle 0|$. The corresponding differential equation obeyed by ρ^I_{OD} is

$$\frac{d\rho^I_{OD}(t)}{dt} = -\frac{1}{2}\eta\sigma^2\, e^{iH^A t}[b + b^\dagger, [b + b^\dagger, \rho_{OD}(t)]]\, e^{-iH^B t}. \tag{16}$$

Multiplying from the left by $e^{-iH^A u}$ and from the right by $e^{iH^B u}$, we get the differential equation

$$e^{-iH^A u}\frac{d\rho^I_{OD}(t)}{dt}\, e^{iH^B u} = -\frac{1}{2}\eta\sigma^2\, e^{iH^A(t-u)}[b + b^\dagger, [b + b^\dagger, \rho_{OD}(t)]]\, e^{-iH^B(t-u)}. \tag{17}$$

Now take Tr_m of this equation, and use cyclic invariance, to get

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Tr}_m\,\mathrm{e}^{-\mathrm{i}H^A u}\rho^I_{OD}(t)\,\mathrm{e}^{\mathrm{i}H^B u} = -\frac{\eta\sigma^2}{2}\,\mathrm{Tr}_m\left\{[b+b^\dagger,[b+b^\dagger,\mathrm{e}^{-\mathrm{i}H^B(t-u)}\,\mathrm{e}^{\mathrm{i}H^A(t-u)}]]\rho_{OD}(t)\right\}. \tag{18}$$

Taking the adjoint of equation (13) and setting $t\to -t$, we get

$$\mathrm{e}^{\mathrm{i}H^B t}\,\mathrm{e}^{-\mathrm{i}H^A t} = N_t\,\mathrm{e}^{\beta_t b^\dagger}\,\mathrm{e}^{\alpha_t b}, \tag{19}$$

from which we easily calculate that the double commutator in equation (18) is

$$\begin{aligned}[b+b^\dagger,[b+b^\dagger,\ \mathrm{e}^{-\mathrm{i}H^B(t-u)}\,\mathrm{e}^{\mathrm{i}H^A(t-u)}]] &= (\beta_{u-t}-\alpha_{u-t})^2\,\mathrm{e}^{-\mathrm{i}H^B(t-u)}\,\mathrm{e}^{\mathrm{i}H^A(t-u)}\\ &= 4\kappa^2(1-\cos\omega_m(u-t))^2\,\mathrm{e}^{-\mathrm{i}H^B(t-u)}\,\mathrm{e}^{\mathrm{i}H^A(t-u)}.\end{aligned} \tag{20}$$

Substituting this into equation (18), and using cyclic invariance of the trace and equation (15), then gives

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\mathrm{Tr}_m\,\mathrm{e}^{-\mathrm{i}H^A u}\rho^I_{OD}(t)\,\mathrm{e}^{\mathrm{i}H^B u} = -2\eta(\kappa\sigma)^2(1-\cos\omega_m(u-t))^2\,\mathrm{Tr}_m\,\mathrm{e}^{-\mathrm{i}H^A u}\rho^I_{OD}(t)\,\mathrm{e}^{\mathrm{i}H^B u}, \tag{21}$$

which can be immediately integrated to give

$$\mathrm{Tr}_m\,\mathrm{e}^{-\mathrm{i}H^A u}\rho^I_{OD}(t)\,\mathrm{e}^{\mathrm{i}H^B u} = \mathrm{e}^{-2\eta(\kappa\sigma)^2\int_0^t \mathrm{d}v(1-\cos\omega_m(u-v))^2}\,\mathrm{Tr}_m\,\mathrm{e}^{-\mathrm{i}H^A u}\rho^I_{OD}(0)\,\mathrm{e}^{\mathrm{i}H^B u}. \tag{22}$$

Setting $u = t$ in this equation, and using equation (15) and cyclic invariance of the trace together with equation (19), we get

$$\begin{aligned} f &= \mathrm{Tr}_m\,\rho_{OD}(t)\\ &= \mathrm{e}^{-2\eta(\kappa\sigma)^2\int_0^t \mathrm{d}v(1-\cos\omega_m(t-v))^2}\,\mathrm{Tr}_m\,\mathrm{e}^{-\mathrm{i}H^A t}\rho^I_{OD}(0)\,\mathrm{e}^{\mathrm{i}H^B t}\\ &= \mathrm{e}^{-2\eta(\kappa\sigma)^2\int_0^t \mathrm{d}v(1-\cos\omega_m v)^2}\,{}_m\langle 0|N\,\mathrm{e}^{\beta_t b^\dagger}\,\mathrm{e}^{\alpha_t b}|0\rangle_m\\ &= \mathrm{e}^{-\frac{3}{16}\eta\ell^2\left(t-\frac{4}{3}\frac{\sin\omega_m t}{\omega_m}+\frac{\sin 2\omega_m t}{6\omega_m}\right)}\,\mathrm{e}^{-\kappa^2(1-\cos\omega_m t)+\mathrm{i}\kappa^2(\omega_m t-\sin\omega_m t)}.\end{aligned} \tag{23}$$

Finally, taking the absolute value of equation (23), we get for the visibility

$$\nu(t) = \exp[-\kappa^2(1-\cos\omega_m t)]\times\exp\left[-\frac{3}{16}\eta\ell^2\left(t-\frac{4}{3}\frac{\sin\omega_m t}{\omega_m}+\frac{\sin 2\omega_m t}{6\omega_m}\right)\right]. \tag{24}$$

Equations (23) and (24) are the results that we quoted in [7].

4. Solution for the visibility by a stochastic unravelling method

In this section we give an alternate derivation of equation (24), using stochastic methods to solve equation (10). We exploit the property that although equation (8) for the stochastic evolution of the state vector uniquely implies the evolution of equation (10) for the expectation density matrix ρ, this relationship is not one to one: there are in fact an infinite number of different stochastic evolutions (or unravellings) which imply equation (10) for the evolution of their expectations [9]. In particular, a simple calculation using the Itô calculus shows that the *linear* stochastic equation

$$\mathrm{d}|\psi_t\rangle = \left[-\frac{\mathrm{i}}{\hbar}H\,\mathrm{d}t + \mathrm{i}\sqrt{\eta}q\,\mathrm{d}W_t - \frac{\eta}{2}q^2\,\mathrm{d}t\right]|\psi_t\rangle \tag{25}$$

also has equation (10) for the evolution equation for $\rho = E[|\psi_t\rangle\langle\psi_t|]$. This means that, as long as one is interested only in the *statistical* properties of the system—i.e. expectation values like $\mathrm{Tr}_m\,\rho_{OD}(t)$ and the visibility—one can choose freely to work either with the stochastic evolution of equation (8) or with the stochastic evolution of equation (25). Of course, for *individual* realizations of the stochastic process, the two equations (8) and (25) imply radically different dynamics; in particular, equation (8) induces the collapse of the wavefunction, while equation (25) does not. However, for all physical quantities that depend only on the expectation of the density matrix, the two evolutions give the same answer.

Let us then resort to equation (25), since it is linear. According to this equation, the initial state (2) evolves as follows:

$$|\psi_t\rangle = \tfrac{1}{\sqrt{2}}\,\mathrm{e}^{-\mathrm{i}\omega_c t}\left[|0\rangle_A|1\rangle_B\left|\phi_t^0\right\rangle_m + |0\rangle_A|1\rangle_B\left|\phi_t^1\right\rangle_m\right], \tag{26}$$

where the state vectors $\left|\phi_t^0\right\rangle_m$ and $\left|\phi_t^1\right\rangle_m$ satisfy the following stochastic differential equation for the mirror centre of mass[5]:

$$\mathrm{d}\phi_t^n(x) = \left[\frac{\mathrm{i}\hbar}{2M}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\,\mathrm{d}t - \frac{\mathrm{i}M\omega_m^2}{2\hbar}x^2\,\mathrm{d}t + \mathrm{i}ngx\,\mathrm{d}t + \mathrm{i}\sqrt{\eta}x\,\mathrm{d}W_t - \frac{\eta}{2}x^2\,\mathrm{d}t\right]\phi_t^n(x), \tag{27}$$

with $n = 0, 1$, and with the coupling constant $g = G/\sigma$. We now have to find the solution for the initial condition $\left|\phi_0^n\right\rangle_m = |0\rangle_m$.

We take as a trial solution,

$$\phi_t^n(x) = \left(\frac{M\omega_m}{\pi\hbar}\right)^{\frac{1}{4}}\exp\left[-a_t^n x^2 + b_t^n x + c_t^n\right], \tag{28}$$

and by substituting it into equation (27) and using the rules of the Itô calculus, we get the following set of equations for the parameters a_t^n, b_t^n and c_t^n,

$$\begin{aligned}
\mathrm{d}a_t^n &= -\frac{2\mathrm{i}\hbar}{M}\left(a_t^n\right)^2\mathrm{d}t + \frac{\mathrm{i}M\omega_m^2}{2\hbar}\,\mathrm{d}t & a_0^n &= \frac{M\omega_m}{2\hbar},\\
\mathrm{d}b_t^n &= \left[\mathrm{i}ng - \frac{2\mathrm{i}\hbar}{M}b_t^n a_t^n\right]\mathrm{d}t + \mathrm{i}\sqrt{\eta}\,\mathrm{d}W_t & b_0^n &= 0,\\
\mathrm{d}c_t^n &= \frac{\mathrm{i}\hbar}{2M}\left[\left(b_t^n\right)^2 - 2a_t^n\right]\mathrm{d}t & c_0^n &= 0.
\end{aligned} \tag{29}$$

The first two equations can be easily integrated and one gets

$$a_t^n = \frac{M\omega_m}{2\hbar}, \qquad b_t^n = \frac{ng}{\omega_m}\left[1 - \mathrm{e}^{-\mathrm{i}\omega_m t}\right] + \mathrm{i}\sqrt{\eta}\int_0^t \mathrm{e}^{-\mathrm{i}\omega_m(t-s)}\,\mathrm{d}W_s. \tag{30}$$

The factor f previously introduced can be written as

$$f = \int_{-\infty}^{+\infty} E\left[\phi_t^0(x)^*\phi_t^1(x)\right]\mathrm{d}x. \tag{31}$$

We reverse the two operations of computing the statistical average $E[\ldots]$ and of taking the partial trace; the integration over x gives

$$\int_{-\infty}^{+\infty}\phi_t^0(x)^*\phi_t^1(x)\,\mathrm{d}x = \exp\left[\frac{\left(b_t^{0*} + b_t^1\right)^2}{8a_t} + c_t^{0*} + c_t^1\right]. \tag{32}$$

As the final step, we have to take the average of equation (32) with respect to the noise. To this end, we compute the stochastic differential of the exponent, obtaining after some algebra

$$\mathrm{d}\left[\frac{\left(b_t^{0*} + b_t^1\right)^2}{8a_t} + c_t^{0*} + c_t^1\right] = \frac{\mathrm{i}\hbar}{2M\omega_m^2}g^2[1 - \mathrm{e}^{-\mathrm{i}\omega_m t}]\,\mathrm{d}t + \frac{\mathrm{i}\sqrt{\eta}\hbar g}{M\omega_m}z_t\,\mathrm{d}t, \tag{33}$$

[5] We have rewritten the Hamiltonian of equation (1) in terms of the mirror centre-of-mass coordinate x, by re-expressing b and $b^\dagger$ in terms of x and $-\mathrm{i}\hbar\,\mathrm{d}/\mathrm{d}x$.

where z_t is the stochastic process given by the formula

$$z_t = \int_0^t \sin\omega_m(t-s)\,\mathrm{d}W_s. \tag{34}$$

The form of the stochastic integral of equation (34) is such that z_t is a Gaussian stochastic process with zero mean, while the correlation function is

$$K(t,s) = E[z_t z_s] = \int_0^{\min(t,s)} \sin\omega_m(t-u)\sin\omega_m(s-u)\,\mathrm{d}u. \tag{35}$$

Equation (33) shows that, as expected, f is the product of a 'deterministic' part f_D, which does not depend on the noise z_t, and a 'stochastic' part f_S which depends on the noise. The deterministic part gives the result of equation (6),

$$\begin{aligned} f_D &= \exp\left[\frac{\mathrm{i}\hbar}{2M\omega_m^2}g^2\int_0^t(1-\mathrm{e}^{-\mathrm{i}\omega_m s})\,\mathrm{d}s\right] \\ &= \mathrm{e}^{\mathrm{i}\kappa^2(\omega_m t-\sin\omega_m t)}\,\mathrm{e}^{-\kappa^2(1-\cos\omega_m t)}. \end{aligned} \tag{36}$$

We now have to compute the stochastic part,

$$f_S = E\left[\exp\left(\mathrm{i}\frac{\sqrt{\eta}\hbar g}{M\omega_m}\int_0^t z_s\,\mathrm{d}s\right)\right]. \tag{37}$$

One easily recognizes, in the above formula, the definition of the characteristic functional $\Phi[k_t]$ of the Gaussian stochastic process z_t, with $k_t = \sqrt{\eta}\hbar g/M\omega_m$. One then has,

$$\begin{aligned} f_S &= \exp\left[-\frac{\eta}{2}\left(\frac{\hbar g}{M\omega_m}\right)^2\int_0^t \mathrm{d}s_1\int_0^t \mathrm{d}s_2\,K(s_1,s_2)\right] \\ &= \exp\left[-\frac{3}{16}\eta\ell^2\left(t-\frac{4}{3\omega_m}\sin\omega_m t+\frac{1}{6\omega_m}\sin 2\omega_m t\right)\right] \end{aligned} \tag{38}$$

with $\ell = 4\kappa\sigma$ the maximum excursion of the mirror centre of mass in its oscillation. (A derivation of equation (38) directly from the Itô calculus is given in appendix B.) The final result for the visibility $\nu = |f|$ is thus

$$\nu(t) = \exp[-\kappa^2(1-\cos\omega_m t)]\exp\left[-\frac{3}{16}\eta\ell^2\left(t-\frac{4}{3}\frac{\sin\omega_m t}{\omega_m}+\frac{\sin 2\omega_m t}{6\omega_m}\right)\right] \tag{39}$$

as also obtained by the method of section 3.

5. Calculation of the stochasticity parameter from the CSL model

In this section we calculate the stochasticity parameter η appearing in equation (8), in terms of parameters that appear in the CSL model for state vector collapse, which applies to systems of identical particles treated by a field-theoretic approach (for a similar calculation based on properties of the complementary error function, see Ghirardi, Pearle, and Rimini [4], appendix C). The relevant CSL equation, taken from equations (8.23) and (8.24) of the review of Bassi and Ghirardi [6], can be written as

$$\frac{\partial}{\partial t}\langle\mathbf{Q}'|\rho|\mathbf{Q}''\rangle = -\Gamma(\mathbf{Q}',\mathbf{Q}'')\langle\mathbf{Q}'|\rho|\mathbf{Q}''\rangle, \tag{40}$$

where

$$\Gamma(\mathbf{Q}',\mathbf{Q}'') = \frac{1}{2}\gamma\int \mathrm{d}^3x[F(\mathbf{Q}'-\mathbf{x})-F(\mathbf{Q}''-\mathbf{x})]^2, \tag{41}$$

and where

$$F(\mathbf{z}) = \int d^3y\, D(\mathbf{y}) \left(\frac{\alpha}{2\pi}\right)^{\frac{3}{2}} e^{-(\alpha/2)(\mathbf{z}+\mathbf{y})^2}. \tag{42}$$

Letting $\mathbf{d} = \mathbf{Q}' - \mathbf{Q}''$ and using translation invariance and space inversion symmetry, we can rewrite equation (41) as

$$\Gamma(\mathbf{Q}', \mathbf{Q}'') = \frac{1}{2}\gamma \int d^3x [F(\mathbf{x}+\mathbf{d}) - F(\mathbf{x})]^2, \tag{43}$$

so that the Taylor expansion gives for the leading small-displacement term (with summation on i, j understood)

$$\Gamma(\mathbf{Q}', \mathbf{Q}'') \simeq \frac{1}{2}\gamma \int d^3x\, d_i d_j \frac{\partial}{\partial x_i} F(\mathbf{x}) \frac{\partial}{\partial x_j} F(\mathbf{x}). \tag{44}$$

We now use the fact that, acting on the exponential within the integral of equation (42), $\frac{\partial}{\partial z_i}$ is equivalent to $\frac{\partial}{\partial y_i}$, which can be integrated by parts to act on the density D. Since for density distributions with cubic or higher symmetry we expect the coefficient of $d_i d_j$ in equation (44) to be proportional to δ_{ij}, we can extract this coefficient by replacing $d_i d_j$ by $\delta_{ij}\mathbf{d}^2/3$, giving

$$\Gamma(\mathbf{Q}', \mathbf{Q}'') \simeq \tfrac{1}{2}\gamma C \mathbf{d}^2, \tag{45}$$

with the coefficient C given by

$$C = \frac{1}{3}\left(\frac{\alpha}{2\pi}\right)^3 \int d^3y \int d^3w\, \partial_i D(\mathbf{y}) \partial_i D(\mathbf{w}) \int d^3x\, e^{-(\alpha/2)[(\mathbf{x}+\mathbf{y})^2+(\mathbf{x}+\mathbf{w})^2]}. \tag{46}$$

We can now complete the square in the exponent,

$$(\mathbf{x}+\mathbf{y})^2 + (\mathbf{x}+\mathbf{w})^2 = 2\left[\mathbf{x} + \tfrac{1}{2}(\mathbf{y}+\mathbf{w})\right]^2 + \tfrac{1}{2}(\mathbf{y}-\mathbf{w})^2, \tag{47}$$

which allows us to do the $\mathbf{x}$ integration, giving

$$C = \frac{1}{24}\left(\frac{\alpha}{\pi}\right)^{\frac{3}{2}} \int d^3y \int d^3w\, \partial_i D(\mathbf{y}) \partial_i D(\mathbf{w})\, e^{-(\alpha/4)(\mathbf{y}-\mathbf{w})^2}. \tag{48}$$

Let us now assume a cubical volume of uniform density D_0 and side S, so that we can take

$$D(\mathbf{w}) = D_0 \prod_{i=1}^{3} \theta(w_i + S/2)\theta(S/2 - w_i). \tag{49}$$

The three terms summed over i in equation (48) give equal contributions, so we have

$$C = \frac{1}{8} D_0^2 \left(\frac{\alpha}{\pi}\right)^{3/2} I_{12} I_3, \tag{50}$$

with

$$I_{12} = \int_{-S/2}^{S/2} \cdots \int_{-S/2}^{S/2} dy_1\, dy_2\, dw_1\, dw_2\, e^{-(\alpha/4)(y_1-w_1)^2}\, e^{-(\alpha/4)(y_2-w_2)^2}, \tag{51}$$

and with

$$I_3 = \int_{-\infty}^{\infty} dy_3 \int_{-\infty}^{\infty} dw_3 [\delta(y_3 + S/2) - \delta(S/2 - y_3)][\delta(w_3 + S/2) - \delta(S/2 - w_3)]\, e^{-(\alpha/4)(y_3-w_3)^2}. \tag{52}$$

When $S^2\alpha \gg 1$, we can use the fact that the exponentials are sharply peaked to get the approximations

$$I_{12} \simeq S^2 4\pi/\alpha \qquad I_3 \simeq 2, \tag{53}$$

giving

$$C \simeq D_0^2 S^2 \left(\frac{\alpha}{\pi}\right)^{1/2}. \tag{54}$$

This identifies the parameter η appearing as the coefficient of the $[q, [q, \rho]]$ term in the density matrix equation of motion (10) as

$$\eta = \gamma C = \gamma S^2 D_0^2 \left(\frac{\alpha}{\pi}\right)^{1/2}, \tag{55}$$

as used in equation (14) of [7].

As a consistency check, let us use equation (55) to determine the transition regime from quadratic growth of Γ to linear growth. For $|\mathbf{d}|\alpha^{1/2} \gg 1$, we know (see Bassi and Ghirardi [6], p 326) that Γ is given by the formula $\Gamma = \gamma n_{\text{out}} D_0$, with n_{out} the number of nucleons in the displaced cube not lying in the original cube, which is clearly (for a third axis displacement) given by $|\mathbf{d}|S^2 D_0$. So equating $(1/2)\gamma S^2 D_0^2(\alpha/\pi)^{1/2}|\mathbf{d}|^2 = \gamma|\mathbf{d}|S^2 D_0^2$, we find that the transition from quadratic to linear growth occurs at $|\mathbf{d}| = 2(\pi/\alpha)^{1/2}$, which is of the order of the width of the Gaussians and so is reasonable.

6. Discussion

To summarize, we have given details of the calculation of the stochastic reduction in the visibility implied by equations (8)–(10), leading to the visibility formula of equations (24) and (39), as well as details of the calculation of the stochasticity parameter η implied by the CSL model, leading to the formula of equation (55). As already discussed, in the absence of the stochastic reduction, the visibility as given by equation (7) starts at 1 at time $t = 0$, decreases as t increases, and then returns to 1 at $t = 2\pi/\omega_m$, at which point the mirror has completed one period of its oscillation. By contrast, with stochasticity present, we learn from equations (24) and (39) that at time $t = 2\pi/\omega_m$ the mirror visibility is damped by a factor $e^{-\Lambda}$, with

$$\Lambda = (3/16)\eta\ell^2(2\pi/\omega_m). \tag{56}$$

Combining this formula with equation (55), in the CSL model we get

$$\Lambda = (3/16)\gamma S^2 D_0^2 \left(\frac{\alpha}{\pi}\right)^{1/2} \ell^2(2\pi/\omega_m). \tag{57}$$

As shown in [7], which gives a detailed discussion of the physical context, for the parameter values appropriate to the CSL model and the Marshall *et al* experiment, equation (57) gives $\Lambda \sim 0.2 \times 10^{-8}$, indicating that according to the CSL model, coherence is maintained to an accuracy of better than one part in 10^8. Thus the Marshall *et al* experiment is orders of magnitude away from a capability of testing spontaneous collapse models for state vector reduction.

Acknowledgments

The work of SA was supported in part by the Department of Energy under grant no DE-FG02-90ER40542. The work of AB and EI was supported in part by the Istituto Nazionale di Fisica Nucleare. We wish to thank Dik Bouwmeester, Marco Genovese and GianCarlo Ghirardi for helpful conversations.

Appendix A. Baker–Hausdorff formulas

We derive here the Baker–Hausdorff formula of equations (13)–(14). Let us define the unitary evolution operator

$$U = \mathrm{e}^{-\mathrm{i}H^A t}, \tag{A1}$$

and the corresponding interaction picture operator

$$U^I = \mathrm{e}^{\mathrm{i}H^B t} U = \mathrm{e}^{\mathrm{i}H^B t}\, \mathrm{e}^{-\mathrm{i}H^A t}. \tag{A2}$$

The operator U^I obeys the equation of motion

$$\begin{aligned} \frac{\mathrm{d}U^I}{\mathrm{d}t} &= \mathrm{e}^{\mathrm{i}H^B t}\mathrm{i}(H^B - H^A)\,\mathrm{e}^{-\mathrm{i}H^B t}U^I \\ &= \mathrm{e}^{\mathrm{i}H^B t}\mathrm{i}G(b + b^\dagger)\,\mathrm{e}^{-\mathrm{i}H^B t}U^I \\ &= [A(t) + B(t)]U^I, \end{aligned} \tag{A3}$$

where we have defined

$$A(t) = \mathrm{i}G\,\mathrm{e}^{\mathrm{i}H^B t} b\,\mathrm{e}^{-\mathrm{i}H^B t} = \mathrm{i}G\,\mathrm{e}^{-\mathrm{i}\omega_m t} b, \qquad B(t) = \mathrm{i}G\,\mathrm{e}^{\mathrm{i}H^B t} b^\dagger\,\mathrm{e}^{-\mathrm{i}H^B t} = \mathrm{i}G\,\mathrm{e}^{\mathrm{i}\omega_m t} b^\dagger. \tag{A4}$$

These obey the commutators

$$[A(s), A(t)] = [B(s), B(t)] = 0, \qquad [A(s), B(t)] = -G^2 \exp[-\mathrm{i}\omega_m(s - t)], \tag{A5}$$

all of which are c-numbers. Integrating equation (A3) with respect to t, and using $U^I(0) = 1$, we get

$$U^I(t) = T \exp\left[\int_0^t \mathrm{d}s\ (A(s) + B(s))\right], \tag{A6}$$

where T orders later times to the left.

Consider now the operator W defined as

$$W = \exp\left[\int_0^t \mathrm{d}s\, B(s)\right] \exp\left[\int_0^t \mathrm{d}s\, A(s)\right], \tag{A7}$$

which obeys

$$\begin{aligned} \frac{\mathrm{d}W}{\mathrm{d}t} &= \exp\left[\int_0^t \mathrm{d}s\, B(s)\right][B(t) + A(t)] \exp\left[\int_0^t \mathrm{d}s\, A(s)\right] \\ &= \left\{B(t) + \exp\left[\int_0^t \mathrm{d}s\, B(s)\right] A(t) \exp\left[-\int_0^t \mathrm{d}s\, B(s)\right]\right\} W. \end{aligned} \tag{A8}$$

Now for general u we have

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t} \exp\left[\int_0^t \mathrm{d}s\, B(s)\right] A(u) \exp\left[-\int_0^t \mathrm{d}s\, B(s)\right] \\ &\quad = \exp\left[\int_0^t \mathrm{d}s\, B(s)\right][B(t), A(u)] \exp\left[-\int_0^t \mathrm{d}s\, B(s)\right] \\ &\quad = [B(t), A(u)], \end{aligned} \tag{A9}$$

where we have used the fact that the commutator $[B(t), A(u)]$ is a c-number. Integrating on t, this gives

$$\exp\left[\int_0^t \mathrm{d}s\, B(s)\right] A(u) \exp\left[-\int_0^t \mathrm{d}s\, B(s)\right] = A(u) + \int_0^t \mathrm{d}s[B(s), A(u)], \tag{A10}$$

and now setting $u = t$ we get

$$\exp\left[\int_0^t \mathrm{d}s\, B(s)\right] A(t) \exp\left[-\int_0^t \mathrm{d}s\, B(s)\right] = A(t) + \int_0^t \mathrm{d}s[B(s), A(t)]. \tag{A11}$$

Comparing with equation (A8), we have obtained

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \left(A(t) + B(t) + \int_0^t \mathrm{d}s[B(s), A(t)]\right) W, \tag{A12}$$

and comparing this with equations (A3) and (A6) for U^I, we get[6]

$$U^I = W \exp\left(-\int_0^t \mathrm{d}u \int_0^u \mathrm{d}s[B(s), A(u)]\right). \tag{A13}$$

Multiplying equation (A2) from the left by $\mathrm{e}^{-\mathrm{i}H^B t}$ and from the right by $\mathrm{e}^{\mathrm{i}H^B t}$, we then get

$$\begin{aligned}
\mathrm{e}^{-\mathrm{i}H^A t}\,\mathrm{e}^{\mathrm{i}H^B t} &= \mathrm{e}^{-\mathrm{i}H^B t} U^I\,\mathrm{e}^{\mathrm{i}H^B t} \\
&= \mathrm{e}^{-\mathrm{i}H^B t} \exp\left[\int_0^t \mathrm{d}s\, B(s)\right] \mathrm{e}^{\mathrm{i}H^B t}\mathrm{e}^{-\mathrm{i}H^B t} \\
&\quad \times \exp\left[\int_0^t \mathrm{d}s\, A(s)\right] \mathrm{e}^{\mathrm{i}H^B t} \exp\left(-\int_0^t \mathrm{d}u \int_0^u \mathrm{d}s[B(s), A(u)]\right) \\
&= \exp\left[\mathrm{e}^{-\mathrm{i}H^B t} \int_0^t \mathrm{d}s\, B(s)\,\mathrm{e}^{\mathrm{i}H^B t}\right] \exp\left[\mathrm{e}^{-\mathrm{i}H^B t} \int_0^t \mathrm{d}s\, A(s)\,\mathrm{e}^{\mathrm{i}H^B t}\right] \\
&\quad \times \exp\left(-\int_0^t \mathrm{d}u \int_0^u \mathrm{d}s[B(s), A(u)]\right) \\
&= \exp\left[\int_0^t \mathrm{d}s\; \mathrm{i}G\,\mathrm{e}^{\mathrm{i}\omega_m s}\,\mathrm{e}^{-\mathrm{i}H^B t} b^\dagger\,\mathrm{e}^{\mathrm{i}H^B t}\right] \\
&\quad \times \exp\left[\int_0^t \mathrm{d}s\; \mathrm{i}G\,\mathrm{e}^{-\mathrm{i}\omega_m s}\,\mathrm{e}^{-\mathrm{i}H^B t} b\,\mathrm{e}^{\mathrm{i}H^B t}\right] \exp\left(-\int_0^t \mathrm{d}u \int_0^u \mathrm{d}s\; G^2\,\mathrm{e}^{\mathrm{i}\omega_m (s-u)}\right) \\
&= \exp\left[\int_0^t \mathrm{d}s\; \mathrm{i}G\,\mathrm{e}^{\mathrm{i}\omega_m s}\,\mathrm{e}^{-\mathrm{i}\omega_m t} b^\dagger\right] \exp\left[\int_0^t \mathrm{d}s\; \mathrm{i}G\,\mathrm{e}^{-\mathrm{i}\omega_m s}\,\mathrm{e}^{\mathrm{i}\omega_m t} b\right] \\
&\quad \times \exp\left(-\int_0^t \mathrm{d}u \int_0^u \mathrm{d}s\; G^2\,\mathrm{e}^{\mathrm{i}\omega_m (s-u)}\right) \\
&= \exp[\kappa(1 - \mathrm{e}^{-\mathrm{i}\omega_m t}) b^\dagger] \exp[-\kappa(1 - \mathrm{e}^{\mathrm{i}\omega_m t}) b] \exp[-\kappa^2(1 - \mathrm{i}\omega_m t - \mathrm{e}^{-\mathrm{i}\omega_m t})],
\end{aligned} \tag{A14}$$

which is equation (14) of section 3.

Appendix B. Basic Itô calculus formulas

The stochastic differential $\mathrm{d}W_t$ behaves heuristically as a random square root of $\mathrm{d}t$, as expressed in the Itô calculus rules

$$\mathrm{d}W_t^2 = \mathrm{d}t, \qquad \mathrm{d}W_t\,\mathrm{d}t = \mathrm{d}t^2 = 0. \tag{B1}$$

As a consequence of equation (B1), the Leibniz chain rule of the usual calculus is modified to

$$\mathrm{d}(AB) = \mathrm{d}A B + A\,\mathrm{d}B + \mathrm{d}A\mathrm{d}B, \tag{B2}$$

[6] This derivation follows appendix 4A of [10]. However, the final result as given there in equation (4A.10) has the wrong sign in front of the exponent involving the commutator of A and B (the − should be a +).

and thus in differentiating a function $f(A)$, one has

$$\mathrm{d}f(A) = f(A+\mathrm{d}A) - f(A) = f'(A)\,\mathrm{d}A + \tfrac{1}{2}f''(A)(\mathrm{d}A)^2. \tag{B3}$$

These formulae are used in the calculations leading to equations (29) and (33) of section 4.

The Itô differential $\mathrm{d}W_t$ is statistically independent of the random process up to time t, so we have the definition

$$E[\mathrm{d}W_t C(t)] = 0 \tag{B4}$$

for any stochastic process $C(t)$ constructed from $\mathrm{d}W_s$ with $s \leqslant t$. From equations (B1)–(B4), we get useful formulae for expectations of integrals. Consider first

$$f(t) = E\left[\int_0^t \mathrm{d}W_u A(u) \int_0^t \mathrm{d}W_u B(u)\right], \tag{B5}$$

which has the differential

$$\begin{aligned} \mathrm{d}f(t) &= E\left[\mathrm{d}W_t A(t) \int_0^t \mathrm{d}W_u B(u) \right.\\ &\quad \left. + \left(\int_0^t \mathrm{d}W_u A(u)\right) \mathrm{d}W_t B(t) + A(t)B(t)\,\mathrm{d}t\right] \\ &= E[A(t)B(t)]\,\mathrm{d}t, \end{aligned} \tag{B6}$$

which integrates back to give

$$E\left[\int_0^t \mathrm{d}W_u A(u) \int_0^t \mathrm{d}W_u B(u)\right] = \int_0^t \mathrm{d}u\, E[A(u)B(u)], \tag{B7}$$

a formula called the Itô isometry. When $A(u)$ and $B(u)$ have differing domains of support D_A and D_B, the integral on the right-hand side of equation (B7) clearly extends only over the intersection $D_A \cap D_B$. Applying equation (B7) to the definition of z_t in equation (34) immediately gives the formula for the correlation function $K(t,s)$ of equation (35). Consider next the expectation

$$f(t) = E\left[\exp\left(\int_0^t \Phi(u,v)\,\mathrm{d}W_v\right)\right]. \tag{B8}$$

Its differential is, by equation (B3),

$$\begin{aligned} \mathrm{d}f &= E\left[\exp\left(\int_0^t \Phi(u,v)\,\mathrm{d}W_v\right)\left(\Phi(u,t)\,\mathrm{d}W_t + \frac{1}{2}\Phi(u,t)^2\,\mathrm{d}t\right)\right] \\ &= \frac{1}{2} f(t)\Phi(u,t)^2\,\mathrm{d}t, \end{aligned} \tag{B9}$$

which integrates back to give

$$E\left[\exp\left(\int_0^t \Phi(u,v)\,\mathrm{d}W_v\right)\right] = \exp\left(\frac{1}{2}\int_0^t \mathrm{d}v\,\Phi(u,v)^2\right). \tag{B10}$$

In particular, setting $u = t$ we get the useful formula

$$E\left[\exp\left(\int_0^t \Phi(t,v)\,\mathrm{d}W_v\right)\right] = \exp\left(\frac{1}{2}\int_0^t \mathrm{d}v\,\Phi(t,v)^2\right). \tag{B11}$$

As an application of equation (B11), consider the expectation $g(t) = E\big[\exp\big(C\int_0^t z_s \mathrm{d}s\big)\big]$, with z_t given by equation (34). Since

$$\begin{aligned} \int_0^t z_s\,\mathrm{d}s &= \int_0^t \mathrm{d}s \int_0^s \sin\omega_m(s-v)\,\mathrm{d}W_v \\ &= \int_0^t \mathrm{d}W_v \int_v^t \mathrm{d}s\,\sin\omega_m(s-v) \\ &= \int_0^t \omega_m^{-1}[1-\cos\omega_m(t-v)]\,\mathrm{d}W_v, \end{aligned} \tag{B12}$$

the expectation $g(t)$ has the form of equation (B11), with $\Phi(t,v) = C\omega_m^{-1}[1-\cos\omega_m(t-v)]$, and we have

$$g(t) = \exp\left(\frac{C^2}{2\omega_m^2}\int_0^t [1-\cos\omega_m(t-v)]^2\,\mathrm{d}v\right), \tag{B13}$$

which corresponds to the integral appearing in equation (23). An alternative expression for $g(t)$ is obtained by using the formula $\Phi(t,v) = C\int_v^t \mathrm{d}s\,\sin\omega_m(s-v)$, which gives

$$\begin{aligned}\int_0^t \mathrm{d}v\,\Phi(t,v)^2 &= C^2\int_0^t \mathrm{d}v\int_v^t \mathrm{d}s_1\int_v^t \mathrm{d}s_2\,\sin\omega_m(s_1-v)\sin\omega_m(s_2-v)\\ &= C^2\int_0^t \mathrm{d}s_1\int_0^t \mathrm{d}s_2\int_0^{\min(s_1,s_2)} \mathrm{d}v\,\sin\omega_m(s_1-v)\sin\omega_m(s_2-v)\\ &= C^2\int_0^t \mathrm{d}s_1\int_0^t \mathrm{d}s_2 K(s_1,s_2),\end{aligned} \tag{B14}$$

with K the correlation function defined in equation (35). When substituted into equation (B11), this corresponds to the integral appearing in equation (38).

References

[1] Marshall W, Simon C, Penrose R and Bouwmeester D 2003 *Phys. Rev. Lett.* **91** 130401

[2] Penrose R 1996 *Gen. Rel. Grav.* **28** 581
Penrose R 2000 *Mathematical Physics 2000* ed A Fokas *et al* (London: Imperial College)

[3] Ghirardi G C, Rimini A and Weber T 1986 *Phys. Rev.* D **34** 470

[4] Pearle P 1989 *Phys. Rev.* A **39** 2277
Ghirardi G C, Pearle P and Rimini A 1990 *Phys. Rev.* A **42** 78

[5] Diósi L 1989 *Phys. Rev.* A **40** 1165
Gatarek D and Gisin N 1991 *J. Math. Phys.* **32** 2152
Chruściński D and Staszewski P 1992 *Phys. Scr.* **45** 193
Bassi A in preparation

[6] For recent reviews of collapse models and related ones, and surveys of the literature, including references to seminal papers by Diósi, Gisin, and Percival, see Bassi A and Ghirardi G C 2003 *Phys. Rep.* **379** 257
Adler S L *Quantum Theory as an Emergent Phenomenon* (Cambridge: Cambridge University Press) chapter 6

[7] Bassi A, Ippoliti E and Adler S L 2005 Towards quantum superpositions of a mirror: an exact open systems analysis *Phys. Rev. Lett.* **94** 030401

[8] Law C K 1994 *Phys. Rev.* A **49** 433
Law C K 1995 *Phys. Rev.* A **51** 2537
Mancini S and Tombesi P 1994 *Phys. Rev.* A **49** 4055
Mancini S, Man'ko V I and Tombesi P 1997 *Phys. Rev.* A **55** 3042
Bose S, Jacobs K and Knight P L 1997 *Phys. Rev.* A **56** 4175
Bose S, Jacobs K and Knight P L 1999 *Phys. Rev.* **59** 3204
Pace A F, Collett M J and Walls D F 1993 *Phys. Rev.* A **47** 3173
Mancini S, Giovannetti V, Vitali D and Tombesi P 2002 *Phys. Rev. Lett.* **88** 120401 see also references therein

[9] Bassi A 2003 *Phys. Rev.* A **67** 62101

[10] Gardiner C W and Zoller P 2000 *Quantum Noise* (Berlin: Springer) pp 128–9